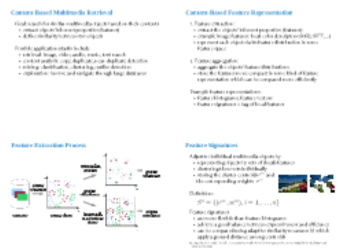
Similarity Matrix Compression for Efficient Signature Quadratic Form Distance Computation

Christian Beecks, Merih Seran Uysal, Thomas Seidl
Data Management and Data Exploration Group
RWTH Aachen University, Germany

International Conference on Similarity Search and Applications
19th September 2010, Istanbul, Turkey

Preliminary Information



Signature Quadratic Form Distance

Similarity Matrix Compression

Experimental Evaluation and Conclusions

Thank you for your attention.

Questions ?

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beecks@cs.rwth-aachen.de

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- extract objects' inherent properties (features)
- define similarity between two objects

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- content analysis: copy, duplicate, near-duplicate detection
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- exploration: browse and navigate through large databases

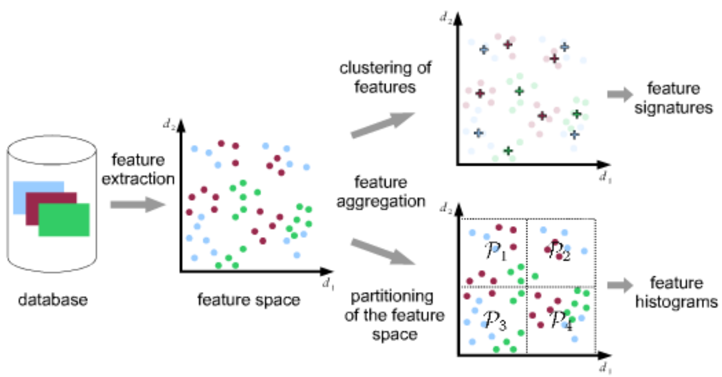
Content-Based Feature Representation

1. Feature extraction:
   - extract the objects' inherent properties (features)
   - example image features: local color descriptors (RGB, SIFT, ...)
   - represent each object via its feature distribution in some feature space

2. Feature aggregation:
   - aggregate the objects' feature distributions
   - store the features more compact in some kind of feature representation which can be compared more efficiently

Example feature representations:
- feature histograms, feature vectors
- feature signatures = bag of local features

Feature Extraction Process



Feature Signatures

Adjust to individual multimedia objects by
- representing objects by sets of (local) features
- clustering these sets individually
- storing the cluster centroids c^{o_i} and the corresponding weights w^{o_i}

Definition:

$$S^o = \{\langle c^{o_i}, w^{o_i}\rangle, i = 1, \ldots, n\}$$

Feature signatures
- are more flexible than feature histograms
- achieve a good balance between expressiveness and efficiency
- can be compared using adaptive similarity measures [1] which apply a ground distance among centroids

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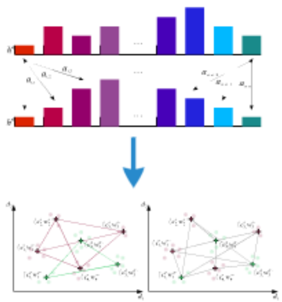
Signature Quadratic Form Distance

Quadratic Form Distance

Idea:

- generalize the concept of Quadratic Form Distance [2,3]

$QFD_A(h^q, h^o) = \sqrt{(h^q - h^o) \cdot A \cdot (h^q - h^o)^T}$



New:

- adapt cross-dimension concept to compare feature signatures
- make use of an inherent similarity function comparing all centroids with each other

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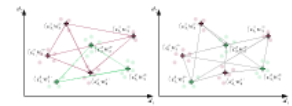
Signature Quadratic Form Distance

Definition:

- given two feature signatures



$$S^q = \{\langle c_i^q, w_i^q \rangle | \; i = 1, \dots, n\}$$

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- a similarity function $f_s(c_i, c_j) \mapsto \mathcal{R}$ comparing centroids
- Signature Quadratic Form Distance [4,5] is defined as follows:

$$SQFD_{f_s}(S^q, S^o) = \sqrt{(w_q | - w_o) \cdot A_{f_s} \cdot (w_q | - w_o)^T}$$

Properties:

- concatenation of weights $(w_q | - w_o) = (w_1^q, \dots, w_n^q, -w_1^o, \dots, -w_m^o)$
- similarity matrix is determined dynamically
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Example

- given the following feature signatures:

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$$\begin{array}{cccccccc} & & \binom{3}{3} & \binom{8}{7} & \binom{4}{7} & \binom{9}{5} & \binom{8}{1} & \\ (w_q | - w_o) = & (& 0.5, & 0.5, & -0.5, & -0.25, & -0.25 &) \end{array}$$

- by using $f_s(c_i, c_j) = \frac{1}{1 + L_2(c_i, c_j)}$ we obtain the similarity matrix:

$$A_{f_s} = \begin{pmatrix} 1 & 0.135 & 0.195 & 0.137 & 0.157 \\ 0.135 & 1 & 0.2 & 0.309 & 0.143 \\ 0.195 & 0.2 & 1 & 0.157 & 0.122 \\ 0.137 & 0.309 & 0.157 & 1 & 0.195 \\ 0.157 & 0.143 & 0.122 & 0.195 & 1 \end{pmatrix}$$

- thus the Signature Quadratic Form Distance becomes:

$$\begin{aligned} SQFD_{f_s}(S^q, S^o) &= \sqrt{(w_q | - w_o) \cdot A_{f_s} \cdot (w_q | - w_o)^T} \\ &= 0.808 \, . \end{aligned}$$

Performance Evaluation

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- Efficiency (Computation time in milliseconds):

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- Signature Quadratic Form Distance can outperform state-of-the-art similarity measures

Similarity Matrix Compression

Idea

- Allow feature signatures to share the same centroids
- Use this "global information" to compress the similarity matrix of the Signature Quadratic Form Distance
- Furthermore: use precomputation possibilities
- Result: We are able to compute Signature Quadratic Form Distance more efficiently

Global and Local Components

Split feature signatures into two parts:

- Global components S^q_g and S^p_g storing shared information

$$S^q_g := \{\langle c, w\rangle \in S^q \mid \exists w' \in \mathcal{R}^+ : \langle c, w'\rangle \in S^p\},$$
$$S^p_g := \{\langle c, w\rangle \in S^p \mid \exists w' \in \mathcal{R}^+ : \langle c, w'\rangle \in S^q\},$$

- Local components S^q_l and S^p_l storing individual information

$$S^q_l := S^q \backslash S^q_g,$$
$$S^p_l := S^p \backslash S^p_g.$$

Sorted Weight Vectors

Rearrange the weights of the feature signatures w.r.t. global and local components to obtain the sorted weight vectors:

$$\widetilde{w}^q := (\widetilde{w}^q_g | \widetilde{w}^q_l) := (\widetilde{w}^{q_1}, \ldots, \widetilde{w}^{q_k} | \widetilde{w}^{q_{k+1}}, \ldots, \widetilde{w}^{q_n}),$$
$$\widetilde{w}^p := (\widetilde{w}^p_g | \widetilde{w}^p_l) := (\widetilde{w}^{p_1}, \ldots, \widetilde{w}^{p_k} | \widetilde{w}^{p_{k+1}}, \ldots, \widetilde{w}^{p_m}),$$

under the following conditions:

(i) $\forall i\ 1 \leq i \leq k,\ \exists c.$
$\langle c, \widetilde{w}^{q_i}\rangle \in S^q_g \wedge \langle c, \widetilde{w}^{p_i}\rangle \in S^p_g,$

(ii) $\forall i\ k+1 \leq i \leq n,\ \exists c.\ \langle c, \widetilde{w}^{q_i}\rangle \in S^q_l,$

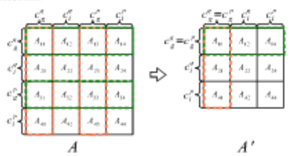
(iii) $\forall i\ k+1 \leq i \leq m,\ \exists c.\ \langle c, \widetilde{w}^{p_i}\rangle \in S^p_l,$

(iv) $\forall i, j\ 1 \leq i, j \leq n,\ i \neq j,\ \exists c, c'.$
$\langle c, \widetilde{w}^{q_i}\rangle \in S^q \wedge \langle c', \widetilde{w}^{q_j}\rangle \in S^q \wedge c \neq c',$

(iv) $\forall i, j\ 1 \leq i, j \leq m,\ i \neq j,\ \exists c, c'.$
$\langle c, \widetilde{w}^{p_i}\rangle \in S^p \wedge \langle c', \widetilde{w}^{p_j}\rangle \in S^p \wedge c \neq c'.$

Similarity Matrix Compression

By making use of centroid structure given by the global components of the sorted weight vectors, we compress the similarity matrix:



and compute the Signature Quadratic Form Distance as:

$$SQFD_A(S^q, S^p) = \sqrt{(\widetilde{w}^q_g - \widetilde{w}^p_g | \widetilde{w}^q_l | - \widetilde{w}^p_l) \cdot A' \cdot (\widetilde{w}^q_g - \widetilde{w}^p_g | \widetilde{w}^q_l | - \widetilde{w}^p_l)^T}$$

Example

Consider the following feature signatures:

$$S^q = \{\langle \binom{2}{2}, 0.5\rangle, \langle \binom{3}{1}, 0.4\rangle, \langle \binom{3}{2}, 0.1\rangle\}$$
$$S^o = \{\langle \binom{2}{2}, 0.3\rangle, \langle \binom{3}{1}, 0.3\rangle, \langle \binom{2}{3}, 0.4\rangle\}$$

and the sorted weight vectors:

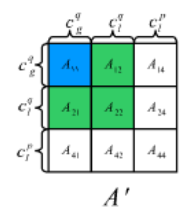
$$\widetilde{w}^q = (0.5,\ 0.4 \mid 0.1)$$
$$\widetilde{w}^o = (0.3,\ 0.3 \mid 0.4)$$

We can compute the Signature Quadratic Form Distance as:

$$SQFD(S^q, S^o) = \sqrt{(0.5 - 0.3,\ 0.4 - 0.3 \mid 0.1 \mid\ - 0.4) \cdot A' \cdot \ldots}$$

Structure of the Compressed Similarity Matrix

- 3x3 block structure according to global and local components:



- Blue block can be computed before a query is given
- Green blocks can be computed once the query is issued

Experimental Evaluation and Conclusions

Experimental Setup

We generated feature signatures by

- varying in the number of centroids: 100 - 800
- setting the centroid dimensionality: 7
- choosing the weights randomly
- changing the ratio c_g of global centroids: 0% - 80%

We computed the Signature Quadratic Form Distance using the similarity function $a_{ij} = e^{-L_2(c_i, c_j)/2}$

Efficiency Evaluation

- Average computation times in milliseconds by using similarity matrix compression with precomputation

	size of feature signatures				
$c_g(\%)$	100	200	400	800	s_f
80	3.14	12.71	50.78	206.05	9.0
60	6.95	27.67	110.45	445.04	4.1
40	11.24	44.93	179.43	720.15	2.6
20	16.09	64.49	257.65	1033.54	1.8
0	28.81	114.96	459.89	1844.17	

- and without precomputation:

	size of feature signatures				
$c_g(\%)$	100	200	400	800	s_f
80	10.37	41.86	166.12	663.7	2.8
60	14.21	56.54	228.09	905.16	2.0
40	18.35	73.76	297.93	1181.04	1.6
20	23.25	93.39	374.31	1493.24	1.2
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Summary

- Signature Quadratic Form Distance is a generalization of the Quadratic Form Distance
- Signature Quadratic Form Distance can compare feature signatures of different size and structure
- Similarity matrix compression uses global information among the feature signatures
- Our approach improves the efficiency of single distance computations by a factor up to 9

Future Work

- So far, we evaluated the similarity matrix compression appraoch only theoretically
- We plan to evaluate the effectiveness of our approach on different multimedia databases
- We plan to incorporate the extraction of global information in the feature extraction process
- Open question: How to index feature signatures according to global components?

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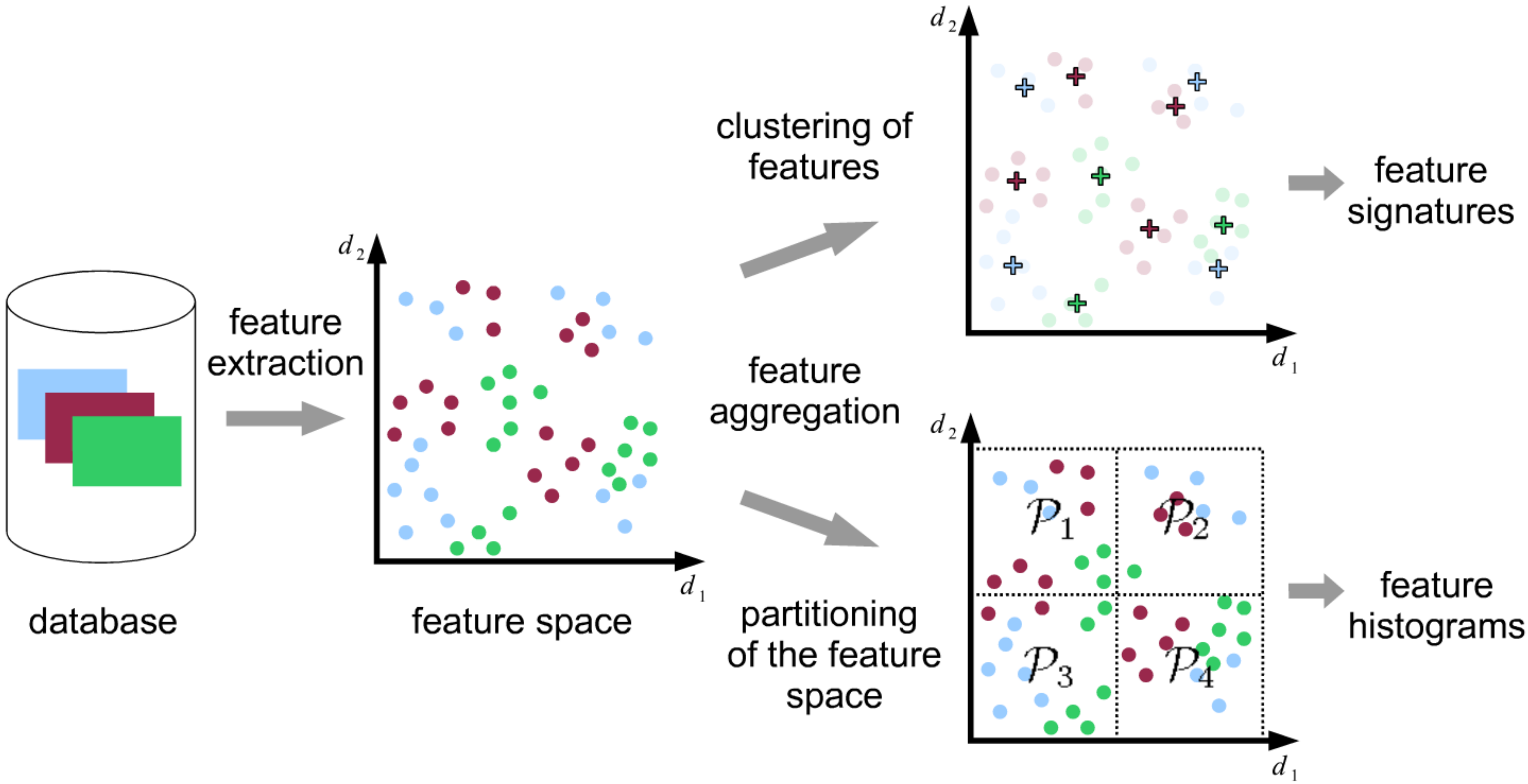

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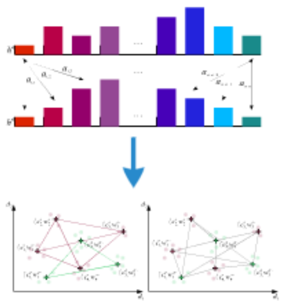

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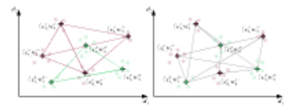

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Properties:

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Performance Evaluation

- Effectiveness (Mean Average Precision):

database	$SQFD_{f_-}$	$SQFD_{f_h}$	$SQFD_{f_g}$	HD	PMHD	WCD	EMD	α_{f_h}	α_{f_g}
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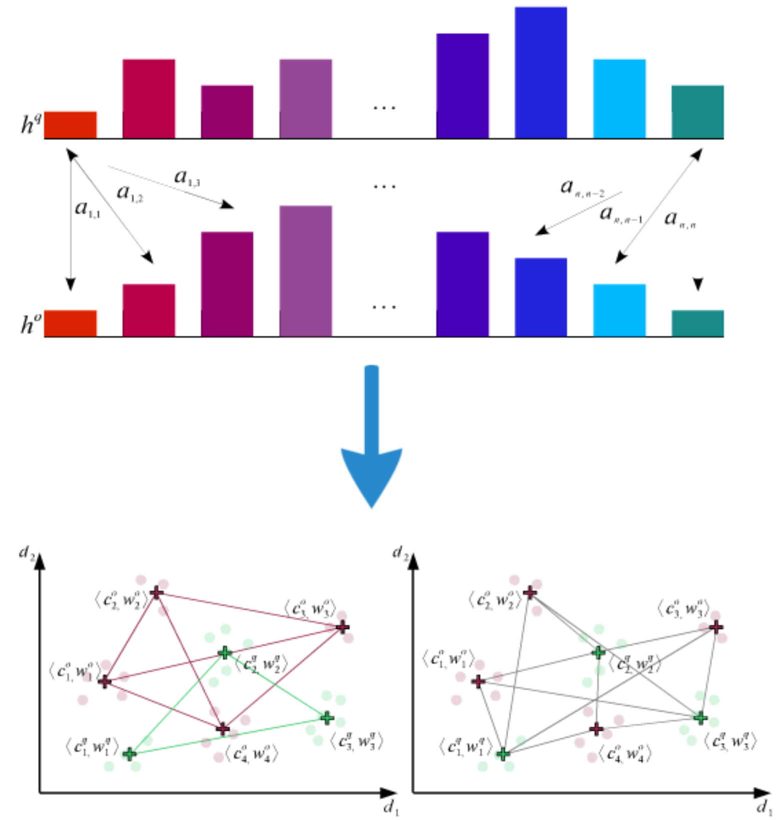

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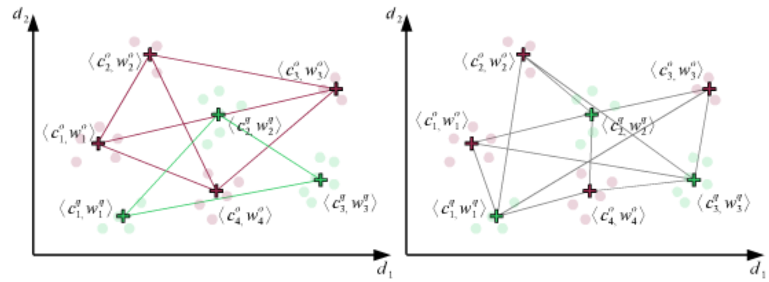

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- Effectiveness (Mean Average Precision):

database	$SQFD_{f_-}$	$SQFD_{f_h}$	$SQFD_{f_g}$	HD	PMHD	WCD	EMD	α_{f_h}	α_{f_g}
Wang	0.592	0.598	**0.613**	0.308	0.476	0.591	0.598	2.7	0.9
Coil100	0.726	0.721	**0.776**	0.425	0.606	0.726	0.710	2.7	0.6
MIR Flickr	0.336	0.338	**0.343**	0.307	0.322	0.335	0.333	2.5	0.6
101objects	0.117	0.128	0.139	0.072	0.105	0.117	**0.141**	1.4	1.6
average:	0.443	0.446	**0.468**	0.278	0.377	0.442	0.446		

- Efficiency (Computation time in milliseconds):

database	size	$SQFD_{f_-}$	$SQFD_{f_h}$	$SQFD_{f_g}$	HD	PMHD	WCD	EMD
Wang	1000	458.9	565.1	1127.0	136.0	234.4	615.3	4880.9
Coil100	7200	1981.5	2422.9	4753.8	631.3	1084.6	2591.2	19393.2
MIR Flickr	25000	9254.7	11399.0	22821.1	2701.5	4635.6	12572.2	73182.9
101objects	9196	3443.2	4223.6	8434.9	1019.5	1749.7	4608.2	33226.7
average:		3784.6	4652.6	9284.2	1122.0	1926.1	5096.7	32670.9

- Signature Quadratic Form Distance can outperform state-of-the-art similarity measures

Similarity Matrix Compression

Idea

- Allow feature signatures to share the same centroids
- Use this "global information" to compress the similarity matrix of the Signature Quadratic Form Distance
- Furthermore: use precomputation possibilities
- Result: We are able to compute Signature Quadratic Form Distance more efficiently

Global and Local Components

Split feature signatures into two parts:

- Global components S_g^q and S_g^p storing shared information

$$\begin{aligned} S_g^q &:= \{\langle c, w\rangle \in S^q \mid \exists w' \in \mathcal{R}^+ : \langle c, w'\rangle \in S^p\}, \\ S_g^p &:= \{\langle c, w\rangle \in S^p \mid \exists w' \in \mathcal{R}^+ : \langle c, w'\rangle \in S^q\}, \end{aligned}$$

- Local components S_l^q and S_l^p storing individual information

$$\begin{aligned} S_l^q &:= S^q \backslash S_g^q, \\ S_l^p &:= S^p \backslash S_g^p. \end{aligned}$$

Sorted Weight Vectors

Rearrange the weights of the feature signatures w.r.t. global and local components to obtain the sorted weight vectors:

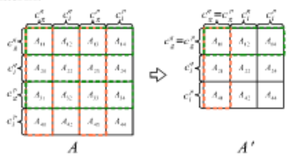
$$\begin{aligned} \widetilde{w}^q &:= (\widetilde{w}_g^q | \widetilde{w}_l^q) := (\widetilde{w}^{q_1}, \ldots, \widetilde{w}^{q_k} | \widetilde{w}^{q_{k+1}}, \ldots, \widetilde{w}^{q_n}), \\ \widetilde{w}^p &:= (\widetilde{w}_g^p | \widetilde{w}_l^p) := (\widetilde{w}^{p_1}, \ldots, \widetilde{w}^{p_k} | \widetilde{w}^{p_{k+1}}, \ldots, \widetilde{w}^{p_m}), \end{aligned}$$

under the following conditions:

$$\begin{aligned} &(i) && \forall i\ 1 \le i \le k,\ \exists c. \\ & && \langle c, \widetilde{w}^{q_i}\rangle \in S_g^q \wedge \langle c, \widetilde{w}^{p_i}\rangle \in S_g^p, \\ &(ii) && \forall i\ k+1 \le i \le n,\ \exists c.\ \langle c, \widetilde{w}^{q_i}\rangle \in S_l^q, \\ &(iii) && \forall i\ k+1 \le i \le m,\ \exists c.\ \langle c, \widetilde{w}^{p_i}\rangle \in S_l^p, \\ &(iv) && \forall i, j\ 1 \le i, j \le n,\ i \ne j,\ \exists c, c'. \\ & && \langle c, \widetilde{w}^{q_i}\rangle \in S^q \wedge \langle c', \widetilde{w}^{q_j}\rangle \in S^q \wedge c \ne c', \\ &(iv) && \forall i, j\ 1 \le i, j \le m,\ i \ne j,\ \exists c, c'. \\ & && \langle c, \widetilde{w}^{p_i}\rangle \in S^p \wedge \langle c', \widetilde{w}^{p_j}\rangle \in S^p \wedge c \ne c'. \end{aligned}$$

Similarity Matrix Compression

By making use of centroid structure given by the global components of the sorted weight vectors, we compress the similarity matrix:



and compute the Signature Quadratic Form Distance as:

$$SQFD_A(S^q, S^p) = \sqrt{(\widetilde{w}_g^q - \widetilde{w}_g^p | \widetilde{w}_l^q | - \widetilde{w}_l^p) \cdot A' \cdot (\widetilde{w}_g^q - \widetilde{w}_g^p | \widetilde{w}_l^q | - \widetilde{w}_l^p)^T}$$

Example

Consider the following feature signatures:

$$S^q = \{\langle \binom{2}{2}, 0.5\rangle, \langle \binom{3}{1}, 0.4\rangle, \langle \binom{3}{2}, 0.1\rangle\}$$

$$S^o = \{\langle \binom{2}{2}, 0.3\rangle, \langle \binom{3}{1}, 0.3\rangle, \langle \binom{2}{3}, 0.4\rangle\}$$

and the sorted weight vectors:

$$\widetilde{w}^q = (0.5,\ 0.4 \mid 0.1)$$

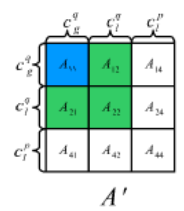
$$\widetilde{w}^o = (0.3,\ 0.3 \mid 0.4)$$

We can compute the Signature Quadratic Form Distance as:

$$SQFD(S^q, S^o) = \sqrt{(0.5 - 0.3,\ 0.4 - 0.3 \mid 0.1 \mid\ - 0.4) \cdot A' \cdot \ldots}$$

Structure of the Compressed Similarity Matrix

- 3x3 block structure according to global and local components:



- Blue block can be computed before a query is given
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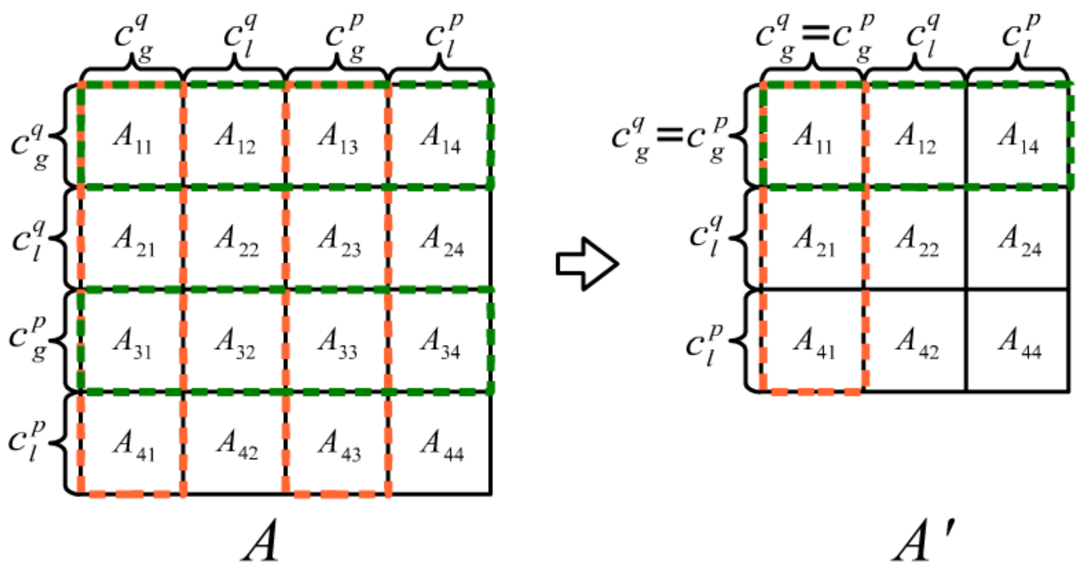

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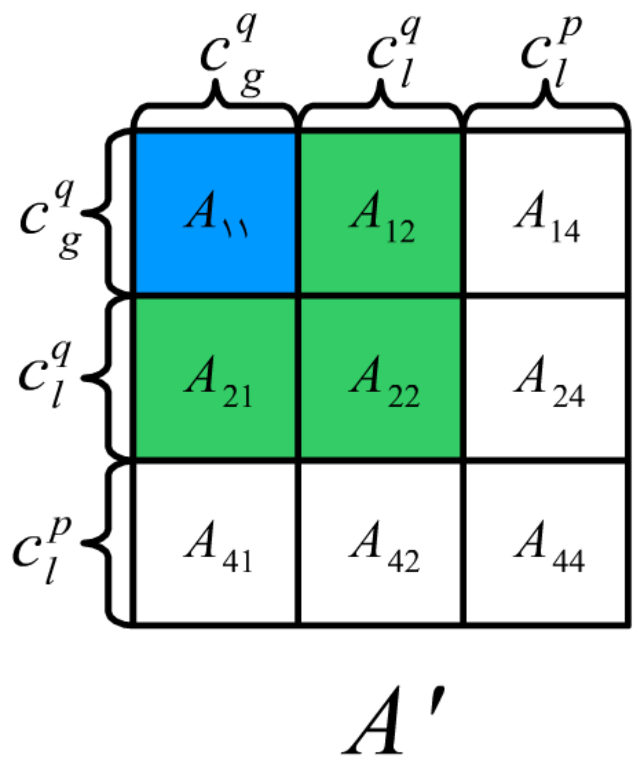

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Experimental Evaluation and Conclusions

Experimental Setup

We generated feature signatures by

- varying in the number of centroids: 100 - 800
- setting the centroid dimensionality: 7
- choosing the weights randomly
- changing the ratio c_g of global centroids: 0% - 80%

We computed the Signature Quadratic Form Distance using the similarity function $a_{ij} = e^{-L_2(c_i,c_j)/2}$

Efficiency Evaluation

- Average computation times in milliseconds by using similarity matrix compression with precomputation

	size of feature signatures				
$c_g(\%)$	100	200	400	800	s_f
80	3.14	12.71	50.78	206.05	9.0
60	6.95	27.67	110.45	445.04	4.1
40	11.24	44.93	179.43	720.15	2.6
20	16.09	64.49	257.65	1033.54	1.8
0	28.81	114.96	459.89	1844.17	

- and without precomputation:

	size of feature signatures				
$c_g(\%)$	100	200	400	800	s_f
80	10.37	41.86	166.12	663.7	2.8
60	14.21	56.54	228.09	905.16	2.0
40	18.35	73.76	297.93	1181.04	1.6
20	23.25	93.39	374.31	1493.24	1.2
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Summary

- Signature Quadratic Form Distance is a generalization of the Quadratic Form Distance
- Signature Quadratic Form Distance can compare feature signatures of different size and structure
- Similarity matrix compression uses global information among the feature signatures
- Our approach improves the efficiency of single distance computations by a factor up to 9

Future Work

- So far, we evaluated the similarity matrix compression appraoch only theoretically
- We plan to evaluate the effectiveness of our approach on different multimedia databases
- We plan to incorporate the extraction of global information in the feature extraction process
- Open question: How to index feature signatures according to global components?

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Thank you for your attention.

Questions ?

Contact: Christian Beecks
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