Indexing inexact proximity search with distance regression in pivot space

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The problem

*Our task*: Speed up proximity search in cases where:
- Distance calculation is expensive.
- Distance-based indexing is needed, because the contents of the data objects cannot be used in the index.
- Some search inexactness is acceptable, meaning we are allowed to trade some search accuracy in return for reduced search computation cost.

*Our contribution*: A new indexing scheme that in some cases provides better computation / accuracy trade-offs than the competition. It also has some drawbacks.
Related work


Takeaway:

- How to perform inexact search by ordering the database according to *promise value* function.
- A specific promise value function, which we use as a baseline in our experiments.
- Experimental setup.
Pivot space

Pivot set:
\[ \mathbb{P} = (p_1, p_2, \ldots, p_m) \]

Mapping object \( o \) to pivot space.
\[ \Phi(o) = (d(o, p_1), d(o, p_2), \ldots, d(o, p_m)) \]
The baseline: Permutation based promise values

The promise value for indexed object $u$ with respect to query $q$ is the correlation (rank correlation coefficient, Spearman’s $\rho$) between the ordering permutation of $\Phi(u)$ and that of $\Phi(q)$. 
What makes a good promise value function?
Two ideas for promise value functions

Distance estimate

\[ \hat{d}(u, q) \]

Probability of inclusion

\[ \Pr(d(u, q) \leq r) \]
Uncertainty in distance estimates

\[ \hat{d}(u, q) \]

distance to query

probability density
Should red or blue object be visited first?

distance to query

probability density

$r$
Should red or blue object be visited first?
Linear model of distance for indexed object $u$

\[ d(u, q) = \beta_{\langle u,0 \rangle} + \sum_{i=1}^{m} \beta_{\langle u,i \rangle} d(q, p_i) + \epsilon_u, \]
Regression-based index (one model per object!)

With \( n \) objects to index and \( m \) pivots, an \( n \times (m + 1) \) matrix:

\[
\begin{pmatrix}
\hat{\beta}_{\langle u_1,0 \rangle}, \hat{\beta}_{\langle u_1,1 \rangle}, \ldots, \hat{\beta}_{\langle u_1,m \rangle} \\
\hat{\beta}_{\langle u_2,0 \rangle}, \hat{\beta}_{\langle u_2,1 \rangle}, \ldots, \hat{\beta}_{\langle u_2,m \rangle} \\
\vdots \\
\hat{\beta}_{\langle u_n,0 \rangle}, \hat{\beta}_{\langle u_n,1 \rangle}, \ldots, \hat{\beta}_{\langle u_n,m \rangle}
\end{pmatrix}
\]

- The coefficients can be discretized to save space.
- Plus \( 2n + 2 \) additional values if we want probabilities.
Building the index

1. Select $n'$ training queries from the objects to be indexed.
2. For each training query $q'$, calculate $\Phi(q')$.
3. For each object to be indexed $u$:
   3.1 For each training query $q'$, calculate $d(u, q')$.
   3.2 Solve the least squares linear regression problem to find the $m + 1$ coefficients $\beta_u$.
   3.3 Store the coefficients in the index.
   3.4 If we want probabilities, also store $\hat{\sigma}_u$, the estimated standard deviation of $\epsilon_u$:

   $$\hat{\sigma}_u = \sqrt{\frac{\sum_{i=1}^{n'}(d(u, q'_i) - \hat{d}(u, q'_i))^2}{n' - m - 1}}$$

4. If we want probabilities for the $k$-NN queries, also store the estimated search radius for each $k$.

(Detail glossed over in this presentation: we exclude $u$ from the training queries used to fit its own model.)
Distance estimates as promise values

\[ \hat{d}(u, q) = \hat{\beta}_{\langle u, 0 \rangle} + \sum_{i=1}^{m} \hat{\beta}_{\langle u, i \rangle} d(q, p_i) \]
Probability-based promise values

\[ r - \hat{d}(u, q) \]
\[ \hat{\sigma}_u \]

- Depends on a lot of assumptions.
- We also ignore the consequence of excluding \( u \) from its own training queries.
Storage costs

With $n$ objects to index and $m$ pivots,

- For distance estimates: $n(m + 1)$ coefficients. (Can be discretized at the cost of some accuracy.)
  - For probabilities: $2n + 2$ additional values.
- Permutation-based index: $nm\lceil\log_2(m)\rceil$ bits in total.
Index building costs

With $n$ objects to index, $m$ pivots and $n'$ training queries,

- Regression-based scheme: $n'(n + m)$ distance calculations plus the solution of $n$ linear regression problems.
- Permutation-based scheme: $nm$ distance calculations, plus some sorting.
Experimental setup

- We borrowed the experimental setup from Chávez, Figueroa & Navarro’s evaluation of the permutation-based scheme.
- Pivots selected randomly.
- Also evaluated versions with pivot set reduced to make storage cost equal to permutation-based index.
- Both synthetic and real-world data sets, but results on real-world data may have more validity.
Evaluating promise value functions: computation / accuracy trade-offs

(Average over many queries.)
Results on normalized edit distance (NED)

(a) 32 pivots

(b) 128 pivots

% retrieved

% of max dist. calc.

Permutations

Probabilities (16 bits unfair)

Distance estimates (16 bits unfair)

Distance estimates (16 bits fair)

Distance estimates (12 bits fair)
Results on documents (TREC)

(a) Range-queries

Permutations
Probabilities (16 bits unfair)
Distance estimates (16 bits fair)
Distance estimates (12 bits fair)

(b) 5-NN queries

% retrieved vs. % of max dist. calc.

% retrieved vs. wall clock time (seconds)
Results on face images (FERET)

(a) % of max distance calculations

(b) Wall clock time (seconds)

- Permutations
- Probabilities (16 bits unfair)
- Distance estimates (16 bits unfair)
- Distance estimates (16 bits fair)
- Distance estimates (12 bits fair)
Why were the probability-based promise values sometimes worse, and never better, than the distance estimates?
Conclusion

Regression-based scheme show some promise, but:

- Takes a lot of time to build the index.
- Vulnerable to deviation from assumptions.